February 06, 2019

Maths Calculation Policy

Introduction

At Priory Rise we believe that in order for children to fully understand how to solve a calculation they need to fully understand the fundamental principles behind the calculation and to achieve this we follow a 'mastery' approach, inspired by the teaching of mathematics in high performing countries internationally and the 'connectionist teaching model'. In order to develop a full understanding of the fundamentals of mathematics, pupils need to be immersed in maths through: concrete experiences and manipulatives; mages and visual representations which demonstrate mathematical thinking; rich discussion and verbal reasoning; and efficient abstract symbols and methods representing their thinking. By exploring concepts in this way, we can ensure that all pupils are capable of success. To reflect our approach to teaching, this calculations policy is split into the following areas:

Concrete

Here you will find explanations of how concepts are represented physically using objects and manipulatives- supporting children in their understanding and reasoning.

Pictorial

In this section will be how calculations are represented visually, either as jottings or as a visual model. Pictorial representations can work as a way to solve a calculation or as a way to help children explain how a formal method works.

Abstract

Abstract representations are when mathematical thinking is shown through symbols., often as a formal written method. If a child can solve a calculation abstractly they should have a deep understanding of the fundamental principles behind a calculation and be able to explain their method in other ways to demonstrate this.

Language

Here you will find examples of how we would expect a child a to verbally explain what they a doing when solving a calculation. Talking is an essential part of learning and allows children to consolidate, embed and deepen their understanding of how to solve a calculation.

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KS1 Addition

In Key Stage 1 pupils are taught to add using concrete resources in order to identify and represent the different parts of a calculation and addition is linked to counting on through the use of number lines. Pupils add one-digit and two-digit numbers to 20, including zero. Much time is spent securing place value of tens and units(ones). Pupils master the concept of exchanging, with ten 'units' being exchanged for one 'ten' where the units digit goes over ten, leading to the introduction on columnar addition in Year 2.

Concrete

Combining 2 numbers and counting the total

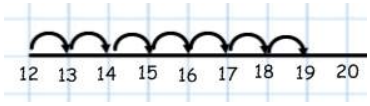
Counting on in 1s

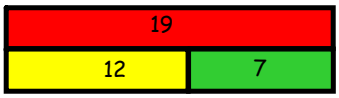
Adding with 'base ten'

By the end of Year 1, children fully understand that 10 units (or ones) are the same as 1 ten.

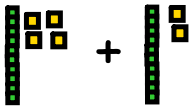
Pictorial

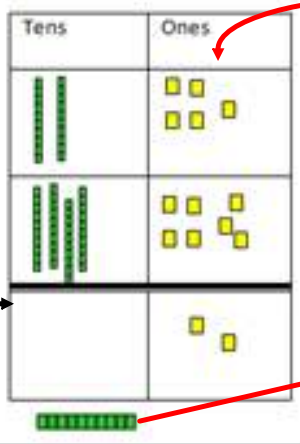
12 + 7 = 19





14 + 12 = 26





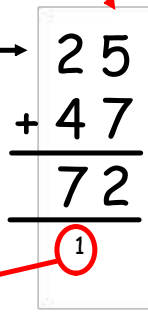
Abstract

12 + 7 = 19

Counting on in 1s mentally

14 + 12 = 26

Combining and counting the total. (Add units first)



Adding the units vertically makes 12. 10 'units' are exchanged for 1 'ten stick' resulting in 2 in the units column and 1 in the tens. Now when we add the tens column there are 7 altogether when we include the ten that has been exchanged.

Language

"If I put 4 and 3 together I now have 7"

"3 more than 6 is 9"

"I started with 16 and added 5 more to make 21 altogether"

"I have 14. To add 12 I need to add 2 units and 1 ten"

"When I add the units I make 12, which means I need to exchange 10 units for 1 ten"

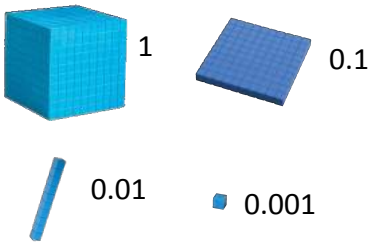
KS2 Addition

The fundamentals of columnar addition are already established by the end of Year 2, so from Year 3 onwards this method is consolidated in order for children to become efficient with adding increasingly larger numbers. Pupils are also expected to now become fluent with the recall of single digit addition facts, allowing the individual calculations in each step to be quickly solved. In Year 4 children are introduced to decimals and connect this with their understanding of place value. As they move into upper KS2, they will begin to include decimal points in the columnar addition method in order to add numbers with up to 3 decimal places.

Concrete

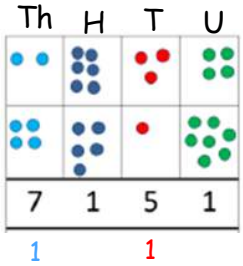
'Place value counters' are used to represent each digit. Children's understanding of place value will now be secure enough to no longer need to see the comparative size of 100s, 10s or 1s.

Decimals



Using base ten equipment to represent a 'unit' being broken down by powers of ten allows children to fully visualise the place value and relative size of each decimal place.

Pictorial



Each column is added from right to left beginning with units. If exchanging is required this is noted underneath. Children can draw a pictorial representation to further support their understanding.

Decimals

13.6 + 5.578 =

Using a pictorial representation ensure children use reasoning to think clearly about the calculation. This bar model allows children to estimate that the answer will be less than 20.

Abstract

```
  2634
+ 4517
  7151
  1  1
```

Decimals

```
  13.600
+  5.578
  19.178
   1
```

Because this number has no hundredths or thousandths, 0 is used as a place holder

It is imperative that every decimal point is in line to ensure the correct digits are added together.

Language

It is important that children maintain an understanding of the overall calculation; for this reason pupils will always refer the relevant place value of each column when adding. E.g. in this example: "4 units add 7 units makes 11 units. 3 tens add 1 ten (and then add the exchanged ten carried over from the units) makes 5 tens. 6 hundreds add 5 hundreds makes 11 hundreds.." and so on.

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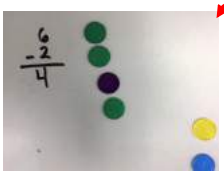
KS1 Subtraction

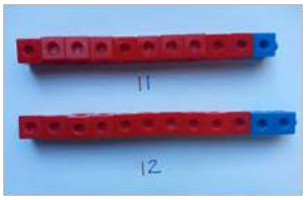
Key Stage 1 pupils are taught that subtraction is the opposite to addition. Using the concrete resources pupils recognise that in subtraction the number will always get smaller and are able to represent by comparing the numbers and finding the difference, or physically removing the correct number of objects. When using pictorial jottings children can show subtraction by counting back on a number line or by 'crossing through' the subtracted amount. Pupils also use their knowledge of place value to exchange one 'ten' for ten 'units' in order subtract 2 digit numbers.

Concrete



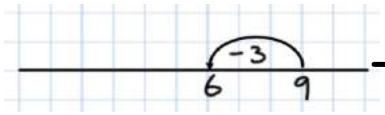
Remove the small number and counting what is left.

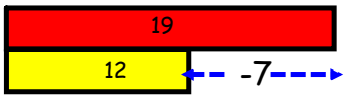
Counting back



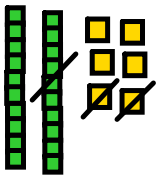
Finding the difference

Pictorial

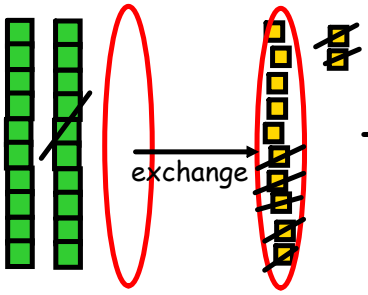




26 - 12 = 14



32 - 15 = 17



Abstract

9 - 3 = 6

Counting back

19 - 7 = 12

Finding the difference

26 - 12 = 14

Subtracting units then tens

$$\begin{array}{r} {}^{2}\cancel{3}\,{}^{1}\cancel{2} \\ -\ 15 \\ \hline 17 \end{array}$$

Exchanging

Language

"There were 7, I took away 3 of them and now there are 4 left"
"The difference between 8 and 6 is 2"
"I have 26 and need to subtract 13 so I'll take away 3 units and 1 ten"
"I have 32 and need to subtract 15... I don't have enough units yet but I can exchange a ten for 10 units"

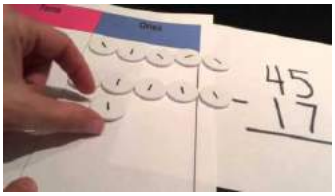
KS2 Subtraction

As with addition, the fundamentals of columnar subtraction are in place by the end of Year 2. From Year 3, pupils work on consolidating their understanding of the method in order and build up to becoming confident in subtracting increasingly larger numbers. Pupils are also expected to now have developed fluency with subtraction facts within 20 so that they efficiently process each calculation. Pupils are introduced to decimals in Year 4 and this is worked into columnar subtraction through their secure understanding place value.

Concrete

Again, 'place value counters' are used to represent each digit.

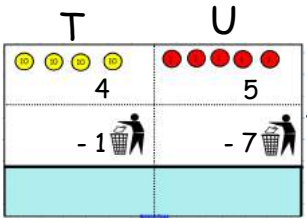
45 - 17 =



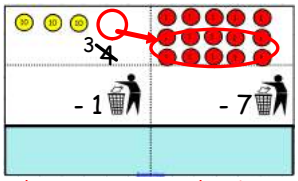
In this example, in order to have enough units to subtract the pupil needed to exchange a ten into units so that they can now work out 15 - 7.

Pictorial

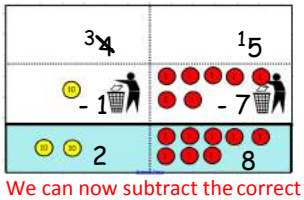
45 - 17 =



The correct representation of 45 is made with counters.



There are not enough units to subtract 7 so a ten counter is exchanged for 10 unit counters to create 30 and 15.



We can now subtract the correct amounts and find the answer (28).

Abstract

$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}5 \\ -\ 1\ 7 \\ \hline 2\ 8 \end{array}$$

Exchanging is noted by striking through.

If there is nothing to exchange, pupils need to go up to the next column, and exchange again (ensuring they record this now as 9)

$$\begin{array}{r} {}^{3}\cancel{2}\,{}^{1}\cancel{4}\,{}^{1}03 \\ -\ 1267 \\ \hline 1136 \end{array}$$

Language

As with addition, it is important that children refer the relevant place value of each column when subtracting. E.g. in this example: "5 *units* subtract 7 *units* doesn't work, so I exchange a ten to make 15 *units* subtract 7 units: that's 8 units. I now have 3 *tens* left, take-away 7 leaves 2 *tens*. The answer is 28."

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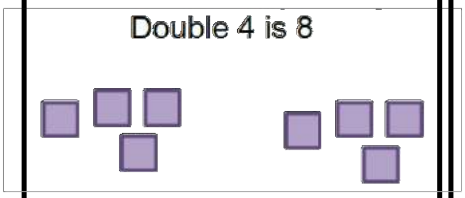
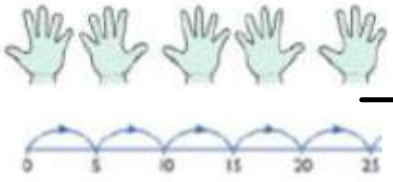
KS1 Multiplication

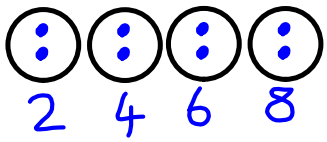
In Key Stage 1 pupils are taught to count in multiples of 2, 5, 10 and 3 and use objects and images to represent this. Pupils are taught to understand multiplication as repeated addition and that the groups must be equal. Pupils make connections between number patterns, arrays and counting. Working practically also demonstrates that multiplication is commutative (can be done in any order.). Pupils learn to double numbers within 20 with concrete and pictorial representations, later using knowledge of place value to double the tens and units separately for a 2 digit number.

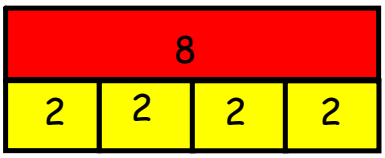
Concrete

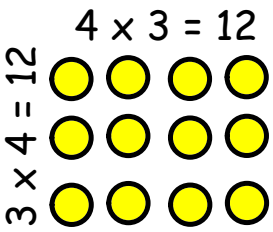


Pictorial









Arrays demonstrate commutativity

Abstract

5, 10, 15, 20, 25

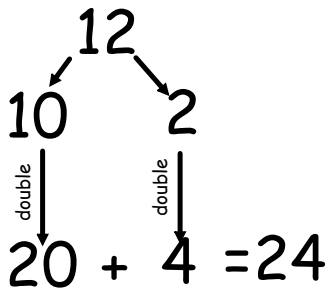
5 x 5 = 25

Counting in groups

2 + 2 + 2 + 2 = 8

4 x 2 = 8

Repeated addition



Partitioning in order to double tens and units

Language

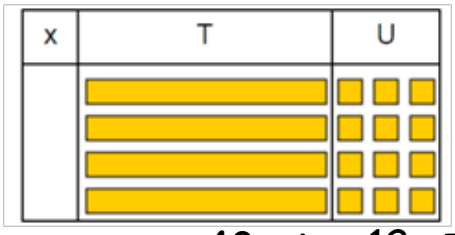
"3 groups of 2 is the same as 6"
"4 lots of 5 makes 20"
"If I count in 10s, after 8 jumps I will get to 80"
"Double 12 is 24"

KS2 Multiplication (1 of 2) - Grid Method

The first formal method pupils will come across for multiplication is the grid method in Year 3. This links well with their experience in using arrays and applies their knowledge of place value in order to accurately multiply. By the end of Year 4 children will be fluent in all times tables facts up to 12 x 12 and this will allow them to solve the individual steps involved quickly.

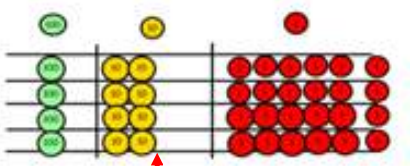
Concrete & Pictorial

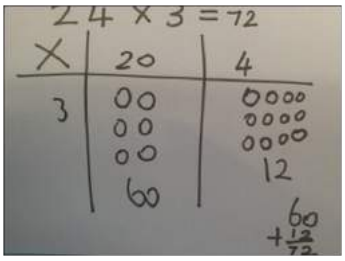
13 x 4 =



40 + 12 = 52

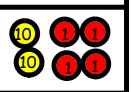
The number being multiplied is partitioned into tens and units. Pupils then multiply each respectively and then add the totals to solve the calculation.





Place value counters are also used and children can draw jottings to represent this.

26 x 14 = 364

	20	6
10		
4		

Children are able to progress to multiplying 2 digit numbers following the same grid in rows. Children work out each box then recombine to find the total.

Abstract

13 x 4 = 52

	10	3
4	40	12

26 x 14 = 364

	20	6
10	200	60
4	80	24

Language

"I can partition 13 in to 10 and 3. 10 times 4 is 40 and 3 times 4 is 12. Altogether, when I recombine them that makes 52."

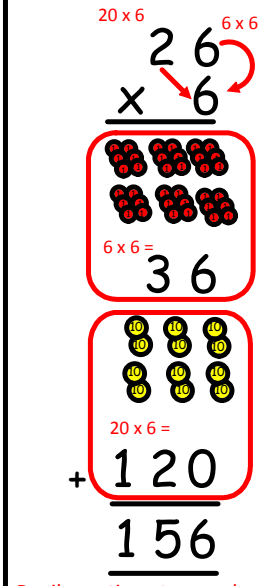
"I have worked out the tens times the tens, the tens times the units, the units times the tens and the units times the units."

KS2 Multiplication (2 of 2) - Columnar

It is expected that pupils learn efficient formal calculation methods and in Year 4 children will be introduced to columnar multiplication. Pupils need to be fluent with their times table facts and be able to think abstractly about multiplication. Pupils begin by using the 'expanded' method, whereby no exchanging of place value will be required and then progress to the compact layout, allowing their method to be more efficient. Long multiplication refers to multiplying by more than 1 digit and it is vital that pupils understand the significance of using 0 as a place holder.

Concrete & Pictorial

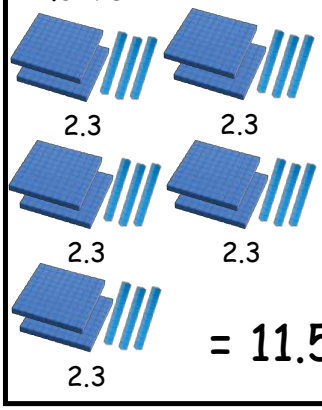
26 x 6 =



Pupils continue to use place value counters to support their understanding. It is vital at this stage the the units are calculated first so that no confusion occurs when exchanging is requited.

Decimals

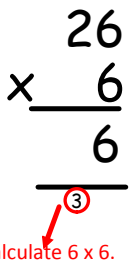
2.3 x 5 =



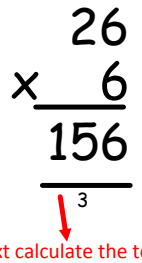
Abstract

Compact

In the compact method the 2 calculations are combined straight away. Pupils may find it easier to think of 20 x 6 as 2 *tens* x 6 = 12 *tens*.

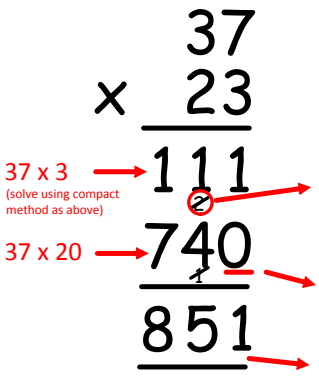


First calculate 6 x 6. The 3 tens are noted underneath the tens column



Next calculate the tens. 6 x 2 = 12 so there are now 12 tens. Add the 3 that were exchanged and there are 15 tens altogether.

Long multiplication



Multiply the top number first by the units of the second number, then by the tens of the second number (and so on right to left if the number has more digits).

37 x 3 (solve using compact method as above)

Pupils are encouraged to strike through any exchanged numbers once they have been added so as not to accidentally add them on at the end.

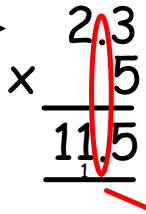
37 x 20

Put in 0 as a place holder first then calculate 37 x 2 (as it is actually 37 x 2 *tens*).

Work out the final answer by adding together the 2 numbers using column addition.

Decimals

2.3 x 5 =



So long as pupils think about columnar multiplication as the top number multiplied by the bottom number, they can work with decimals without changing their method.

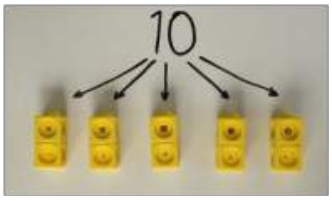
Decimal points must line up.

KS1 Division (1 of 2) - Sharing

Initially, pupils understand division as sharing an amount between a given number of groups and use concrete and pictorial representations to show this. Pupils also learn to halve numbers within 20 by sharing into 2 groups. Conceptual understanding is embedded using concrete and pictorial representations, recognising that only even numbers can be halved. Pupils later use knowledge of place value to halve the tens and units separately for a 2 digit number.

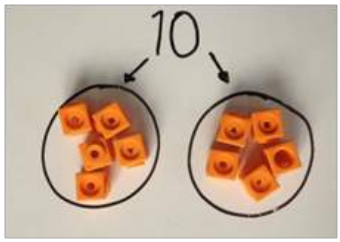
Concrete

10 ÷ 5 =



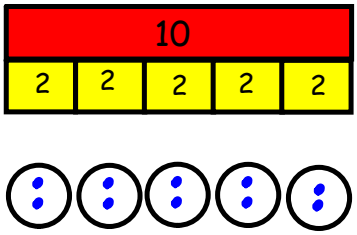
Objects are shared equally, 1 by 1.

Half of 10 =



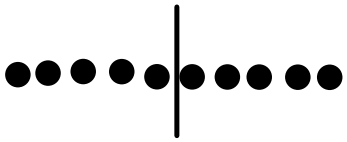
Pictorial

10 ÷ 5 =



Children draw circles and share out dots 1 by 1.

Half of 10 =

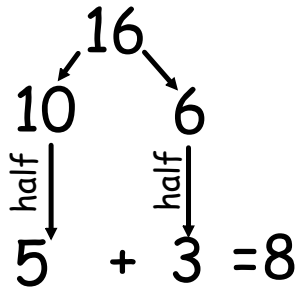


Drawing the number as a line makes it easier for children to identify half without one to one counting.

Abstract

10 ÷ 5 = 2

Half of 16 =



Partition tens and units, then double each.

Language

"I shared 10 sweets with 5 friends and they received 2 each"
"12 shared between 4 is 3"

KS1 Division (2 of 2) - Grouping

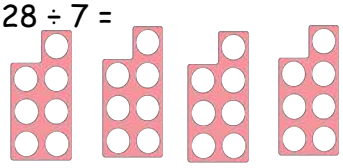
As children begin to deal with larger numbers, they will realise that sharing is not an efficient way of dividing as it requires counting by 1. Like multiplication, pupil's knowledge of counting in multiples of 2, 5, 10 and 3 is used to solve problems involving division by partitioning a number into equal groups. Pupils are then about to count up the number of groups (as opposed to the number within each group). When moving away from concrete representations, children will need to count up in the equal groups, stop counting at the 'target number' and then count how many groups there are.

Concrete

50 ÷ 10 =

Collect groups of 10 until target number of 50 is reached.

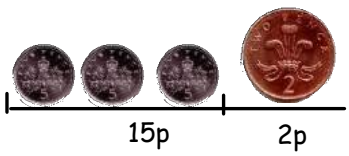
28 ÷ 7 =



7 14 21 28

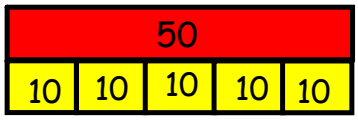
Remainders
(greater depth in Year 2)

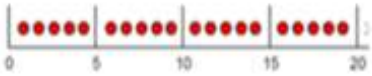
17 ÷ 5 =



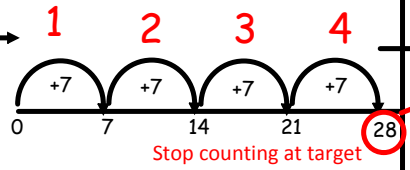
Pictorial

50 ÷ 10 =



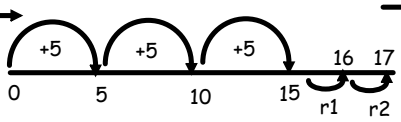


28 ÷ 7 =



Remainders
(greater depth in Year 2)

17 ÷ 5 =



Abstract

50 ÷ 10 = 5

20 ÷ 5 = 4

28 ÷ 7 = 4

Remainders
(greater depth in Year 2)

17 ÷ 5 = 4 r2

Language

"50 cut into groups of 10 makes 5 groups"
"If I count in jumps of 7 to 28 then there are 4 groups 7"
"28 divided by 4 is 7"

KS2 Division (1 of 2) - Grouping

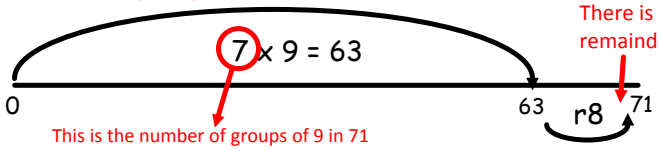
In Key Stage 2, pupils are taught to link division to known multiplication facts. It is therefore essential that times tables facts up to 12 x 12 are known so that division calculations can solved fluently. To solve division problems, pupils think of the closest multiple of the divisor (without going any higher than the target number) to find the number of groups; the remainder is then the difference between the multiple they have found and the target number. The 'bus-stop' division method is used where the number is larger than pupils' known multiplication facts and allows pupils to find the groups of the divisor within each place value column.

Concrete & Pictorial

71 ÷ 9 =

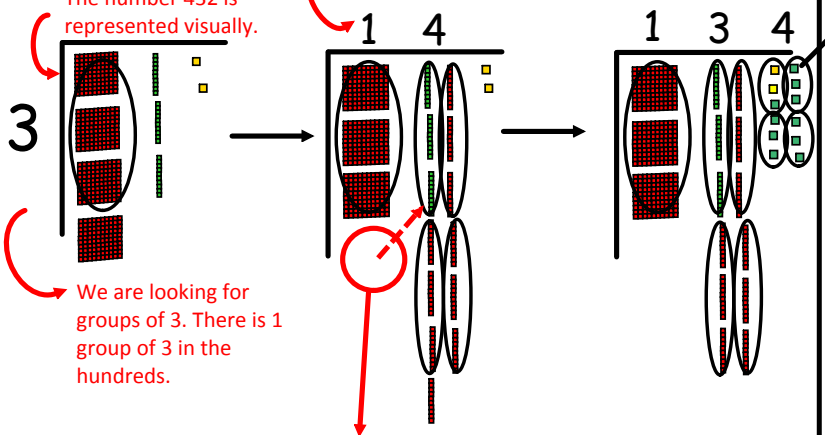
9 is the divisor

In this example, pupils think of their 9 x table and identify the multiple of 9 that is as close to 71 as they can get.



'Bus-stop'

432 ÷ 3 =



Abstract

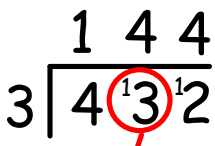
71 ÷ 9

(7 x 9 = 63)

(71 - 63 = 8)

= 7 r8

'Bus-stop'

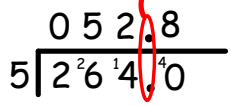


The exchanged hundred is noted as a small 1 above the tens column- now representing 13 tens.

Remainders as decimals

264 ÷ 5 =

Decimal points must be in same place



If there is a remainder, the number can be continued by putting in a decimal point then exchanging each remaining unit for 10 tenths and continuing until the groups fit equally. Here there is a remainder of 4 units and this becomes 40 in the tenths place.

Language

"To divide 71 by 9 I need to think of the 9 times table... I can make 7 groups of 9 because 7 x 9 is 63 and I can't get any closer to 71 without going over. The difference between 63 and 71 is 8 so my answer is 7 remainder 8."

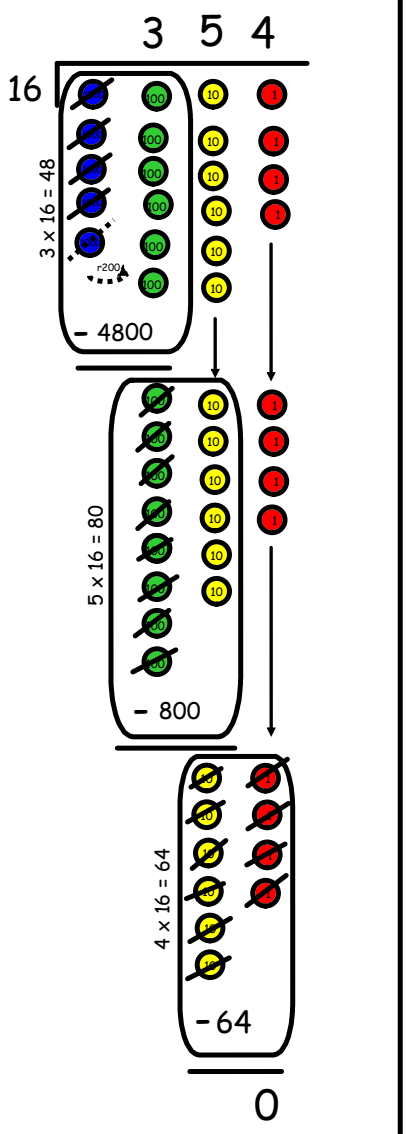
Bus-stop: "I'm looking for groups of 3 in 432... I can make one group of 3 out of the hundreds with one remaining which I'll exchange for 10 tens. I now have 13 tens..."

KS2 Division (2 of 2) - Long Division

In Year 6, children will need to use long division to solve more complicated division problems where the divisor is a 2 or more digit number. Long division is, in essence, an expanded way of representing bus-stop division where each time a multiple of the divisor is found, it is subtracted from the original amount and this is recorded below (rather than exchanging into the new column). Being more methodical in this way allows the calculation to be broken down and the process understood more clearly.

Concrete & Pictorial

5664 ÷ 16 =



Abstract

5664 ÷ 16 =

Key Fact Family

2 x 16 = 32

3 x 16 = 48

4 x 16 = 64

5 x 16 = 80

10 x 16 = 160

Pupils quickly jot down the key multiples of the divisor to help them in each step, a good starting point is 2x, 3x, 4x, 5x, and 10x

```
      3
16 | 5 6 6 4
   - 4 8
     8
```

By looking at the key fact family, we know that 3 x 16 is 48, so 3 groups of 16 fit into 56.

The remainder of 8 is shown by subtracting the 48 from 56.

```
      3 5
16 | 5 6 6 4
   - 4 8
     8 6 4
   - 8 0
       6
```

The 6 tens and 4 units are 'brought down' to show that, after subtracting 4 thousands and 8 hundreds, there is 864 left.

5 x 16 = 80 so 5 groups of 16 are recorded at the top and 80 is subtracted from 86

```
      3 5 4
16 | 5 6 6 4
   - 4 8
     8 6 4
   - 8 0
     6 4
   - 6 4
       0
```

4 x 16 = 64

We have subtracted down to 0 so the division is complete. There are 354 groups of 16 in 5664